Podstawa programowa – technik programista klasa 1

Spis treści

[1. Setup development environment. 2](#_Toc171276812)

[2. Conversion DEC/HEX/OCT/BIN including Horner's scheme. 2](#_Toc171276813)

[2.1 Conversion of a decimal number to a hexadecimal number 2](#_Toc171276814)

[2.2 Conversion of a hexadecimal number to a decimal number 5DE0 2](#_Toc171276815)

[2.3. Horner's scheme used for number conversion 2](#_Toc171276816)

[3. Making Change (Cashier's Problem) 3](#_Toc171276817)

[4. Prime numbers / perfect numbers 3](#_Toc171276818)

[Point 5 Explanation: 4](#_Toc171276819)

[Why is this needed? 4](#_Toc171276820)

[Example: Number 36 4](#_Toc171276821)

[Example: Number 25 (Perfect Square) 5](#_Toc171276822)

[5. Prime factorization 5](#_Toc171276823)

[Definition: 6](#_Toc171276824)

[Steps for Prime Factorization: 6](#_Toc171276825)

[Example: Prime Factorization of 84 6](#_Toc171276826)

[Another Example: Prime Factorization of 120 6](#_Toc171276827)

[General Tips: 7](#_Toc171276828)

[6. Greatest Common Divisor (GCD), Least Common Multiple (LCM) 7](#_Toc171276829)

[7. Solving a Diophantine Equation using extended Euclidean Algorithm 8](#_Toc171276830)

[8. Sieve of Eratosthenes 9](#_Toc171276831)

[Steps of the Sieve of Eratosthenes 9](#_Toc171276832)

[Example: Finding Primes Up to 30 9](#_Toc171276833)

[9. Fast Exponentiation (Exponentiation by Squaring) 10](#_Toc171276834)

[10. Finding Roots - Newton-Raphson Method 11](#_Toc171276835)

[11. Finding the Minimum and Maximum Elements in a Set 11](#_Toc171276836)

[12. Additional content 11](#_Toc171276837)

# Setup development environment.

VS Code or Visual Studio Community

1.1 VS Code

<https://www.youtube.com/watch?v=DMWD7wfhgNY>

download

<https://code.visualstudio.com/docs/?dv=win64user>

follow this instruction:

<https://code.visualstudio.com/docs/cpp/config-mingw>

# Conversion DEC/HEX/OCT/BIN including Horner's scheme.

## 2.1 Conversion of a decimal number to a hexadecimal number

Convert the decimal number 24032 to a hexadecimal number.

24032 / 16 = 1502; remainder = 0

1502 / 16 = 93; remainder = 14 or E

93 / 16 = 5; remainder = 13 or D

5 / 16 = 0; remainder = 5

Reading the remainders from the bottom up, we get the hexadecimal number: 5DE0

## 2.2 Conversion of a hexadecimal number to a decimal number 5DE0

5\**16^3 + 13\**16^2 + 14\*16^1 + 0

Exercise: write a program to convert HEX -> DEC,

consider if the program is universal and if it can be easily modified to work for other bases/number systems (e.g., OCT/BIN). DEC -> HEX HEX -> DEC

## 2.3. Horner's scheme used for number conversion

In Polish

<https://www.youtube.com/watch?v=EY5PnPlWnR0&ab_channel=MaturaInformatyka-Ma%C5%82gorzataPiekarska>

In English

<https://www.youtube.com/watch?v=qDG311jci_0&ab_channel=AhmedAlmansor>

<https://eduinf.waw.pl/inf/alg/006_bin/0003.php>

# Making Change (Cashier's Problem)

In Polish

<https://eduinf.waw.pl/inf/utils/021_2021/1004.php>

in English

<https://www.geeksforgeeks.org/greedy-algorithm-to-find-minimum-number-of-coins/>

# Prime numbers / perfect numbers

Prime numbers

<https://www.geeksforgeeks.org/c-program-to-check-prime-number/>

hint -> sqrt(n)

<https://stackoverflow.com/questions/4424374/determining-if-a-number-is-prime>

Perfect numbers

<https://prepinsta.com/cpp-program/cpp-program-to-check-whether-a-number-is-perfect-number-or-not/>

Badanie pierwszości liczby

<https://home.agh.edu.pl/~zobmat/2021/rzepka_radoslaw/algorytmy.html>

Liczby doskonałe

<https://www.algorytm.edu.pl/algorytmy-maturalne/liczby-doskonale.html>

### Point 5 Explanation:

if(n == p \* p) s -= p;

#### What this does:

If n is a perfect square, the divisor p (which is the square root of n) will be added twice in the loop. To correct this, we subtract p once from the sum s.

### Why is this needed?

When calculating the sum of the divisors of n, each divisor pair (i.e., i and n/i) is added to the sum. For example, for a number like 28:

* Divisors are 1, 2, 4, 7, 14, 28.
* Proper divisors are 1, 2, 4, 7, 14.
* Sum of proper divisors: 1 + 2 + 4 + 7 + 14 = 28 (perfect number).

But if the number is a perfect square, like 36:

* Divisors are 1, 2, 3, 4, 6, 9, 12, 18, 36.
* Proper divisors are 1, 2, 3, 4, 6, 9, 12, 18.
* Pairs of divisors: (1, 36), (2, 18), (3, 12), (4, 9), (6, 6).

Notice here that 6 is a divisor that pairs with itself because 6\*6 = 36. In the loop, 6 would be added twice, so we need to correct this by subtracting it once.

### Example: Number 36

#### Step-by-Step:

1. n = 36
2. Calculate p = sqrt(36) = 6
3. Initialize s = 1 (since 1 is a divisor of all numbers)

#### Loop through possible divisors:

* For i = 2:
  + 36 % 2 == 0 (True), so add 2 + 36/2 = 2 + 18 = 20 to s
  + s = 1 + 20 = 21
* For i = 3:
  + 36 % 3 == 0 (True), so add 3 + 36/3 = 3 + 12 = 15 to s
  + s = 21 + 15 = 36
* For i = 4:
  + 36 % 4 == 0 (True), so add 4 + 36/4 = 4 + 9 = 13 to s
  + s = 36 + 13 = 49
* For i = 5:
  + 36 % 5 == 0 (False), nothing added
* For i = 6:
  + 36 % 6 == 0 (True), so add 6 + 36/6 = 6 + 6 = 12 to s
  + s = 49 + 12 = 61

At this point, we've added the square root divisor 6 twice.

#### Adjust for perfect square:

* Since 36 == 6\*6, subtract one occurrence of 6 from s
  + s = 61 - 6 = 55

### Example: Number 25 (Perfect Square)

1. n = 25
2. Calculate p = sqrt(25) = 5
3. Initialize s = 1

#### Loop through possible divisors:

* For i = 2:
  + 25 % 2 == 0 (False), nothing added
* For i = 3:
  + 25 % 3 == 0 (False), nothing added
* For i = 4:
  + 25 % 4 == 0 (False), nothing added
* For i = 5:
  + 25 % 5 == 0 (True), so add 5 + 25/5 = 5 + 5 = 10 to s
  + s = 1 + 10 = 11

At this point, we've added the square root divisor 5 twice.

#### Adjust for perfect square:

* Since 25 == 5\*5, subtract one occurrence of 5 from s
  + s = 11 - 5 = 6

Now, we correctly sum up the divisors:

* Proper divisors of 25: 1, 5
* Correct sum: 1 + 5 = 6

#### Conclusion:

For perfect squares, the square root is counted twice in the divisor pairs, so we must subtract it once to correct the sum. This adjustment ensures the algorithm accurately determines if n is a perfect number.

# Prime factorization

The prime factorization of a number involves breaking it down into its smallest prime number components. Here’s a detailed explanation and example:

### Definition:

**Prime Factorization**: The process of determining the prime numbers that multiply together to give the original number.

### Steps for Prime Factorization:

1. **Start with the smallest prime number (2)**.
2. **Divide the number by the prime number**.
3. **Continue dividing** until the result is no longer divisible by that prime number.
4. **Move to the next prime number (3, 5, 7, etc.)** and repeat the process.
5. **Stop when the result is 1**.

### Example: Prime Factorization of 84

#### Step-by-Step Process:

1. **Divide by 2 (the smallest prime number)**:
   * 84÷2=4284 \div 2 = 4284÷2=42
   * 84 is divisible by 2, so 2 is a factor.
2. **Continue dividing by 2**:
   * 42÷2=2142 \div 2 = 2142÷2=21
   * 42 is still divisible by 2, so 2 is again a factor.
3. **Move to the next prime number (3)**:
   * 21÷3=721 \div 3 = 721÷3=7
   * 21 is divisible by 3, so 3 is a factor.
4. **Move to the next prime number (7)**:
   * 7÷7=17 \div 7 = 17÷7=1
   * 7 is divisible by 7, so 7 is a factor.

#### Result:

* Prime factors of 84 are: 2×2×3×72 \times 2 \times 3 \times 72×2×3×7
* In exponential form: 22×3×72^2 \times 3 \times 722×3×7

### Another Example: Prime Factorization of 120

1. **Divide by 2**:
   * 120÷2=60120 \div 2 = 60120÷2=60
2. **Continue dividing by 2**:
   * 60÷2=3060 \div 2 = 3060÷2=30
3. **Continue dividing by 2**:
   * 30÷2=1530 \div 2 = 1530÷2=15
4. **Move to the next prime number (3)**:
   * 15÷3=515 \div 3 = 515÷3=5
5. **Move to the next prime number (5)**:
   * 5÷5=15 \div 5 = 15÷5=1

#### Result:

* Prime factors of 120 are: 2×2×2×3×52 \times 2 \times 2 \times 3 \times 52×2×2×3×5
* In exponential form: 23×3×52^3 \times 3 \times 523×3×5

### General Tips:

* **Always start with the smallest prime number (2)**.
* **Check divisibility**: If the number is even, start with 2. If the number ends in 0 or 5, it's divisible by 5, etc.
* **Proceed to the next prime number** if the current prime no longer divides the number.
* **Prime factorization ends** when the quotient is 1.

Prime factorization is fundamental in number theory and has applications in cryptography, computer science, and mathematics.

# Greatest Common Divisor (GCD), Least Common Multiple (LCM)

The greatest common divisor of two integers a and b is the largest integer that divides both of them without leaving a remainder.

The Euclidean algorithm for finding GCD is based on iterative division:

* Repeat the division of the larger number by the smaller number until the remainder is zero.
* The GCD of two numbers is the last non-zero remainder.

function GCD(a, b):

while b ≠ 0:

r = a % b

a = b

b = r

return a

For example, GCD(24, 36) follows these steps:

36 is greater than 24, so divide: 36 mod 24 = 12

Now 24 is greater than 12, so divide again: 24 mod 12 = 0

The last non-zero remainder is 12, so GCD (24, 36) = 12.

The least common multiple of two integers a and b is the smallest integer that is divisible by both of them.

LCM can be calculated using the relationship with GCD:

LCM(a,b) = ∣a⋅b∣ / GCD(a,b)

The Euclidean algorithm can also be used to calculate LCM efficiently:

For example, LCM(24, 36):

* + Calculate GCD(24, 36), which is 12.
  + LCM(24, 36) = ∣24⋅36∣ / 12 = 864 / 12 = 72

The Euclidean algorithm is computationally efficient and widely used for quickly determining GCD and LCM of two integers.

In Polish

<https://www.algorytm.edu.pl/algorytmy-maturalne/algorytm-eulkidesa.html>

in English

<https://www.freecodecamp.org/news/euclidian-gcd-algorithm-greatest-common-divisor/>

# Solving a Diophantine Equation using extended Euclidean Algorithm

The Extended Euclidean Algorithm is a method to find the greatest common divisor (GCD) of two integers and also to express the GCD as a linear combination of these integers. This is particularly useful for solving Diophantine equations of the form:

ax + by = c

where a, b and c are given integers, and x and y are unknown integers that we need to solve for.

*Extended Euclidean algorithm also finds integer coefficients x and y such that: ax + by = gcd(a, b)*

**Examples:**

***Input:****a = 30, b = 20****Output:****gcd = 10, x = 1, y = -1  
(Note that 30\*1 + 20\*(-1) = 10)*

***Input:****a = 35, b = 15****Output:****gcd = 5, x = 1, y = -2  
(Note that 35\*1 + 15\*(-2) = 5)*

The extended Euclidean algorithm updates the results of gcd(a, b) using the results calculated by the recursive call gcd(b%a, a). Let values of x and y calculated by the recursive call be x1 and y1. x and y are updated using the below expressions.

*ax + by = gcd(a, b)  
gcd(a, b) = gcd(b%a, a)  
gcd(b%a, a) = (b%a)x1+ ay1  
ax + by = (b%a)x1+ ay1  
ax + by = (b – [b/a] \* a)x1+ ay1  
ax + by = a(y1 – [b/a] \* x1) + bx1*

*Comparing LHS and RHS,  
x = y1 – b/a \* x1  
 y = x1*

<https://www.geeksforgeeks.org/euclidean-algorithms-basic-and-extended/>

# Sieve of Eratosthenes

The Sieve of Eratosthenes is a classic algorithm used to find all prime numbers up to a given limit nnn. It works by iteratively marking the multiples of each prime number starting from 2.

### Steps of the Sieve of Eratosthenes

1. **Create a list of consecutive integers** from 2 to n:

[2,3,4,5,…,n]

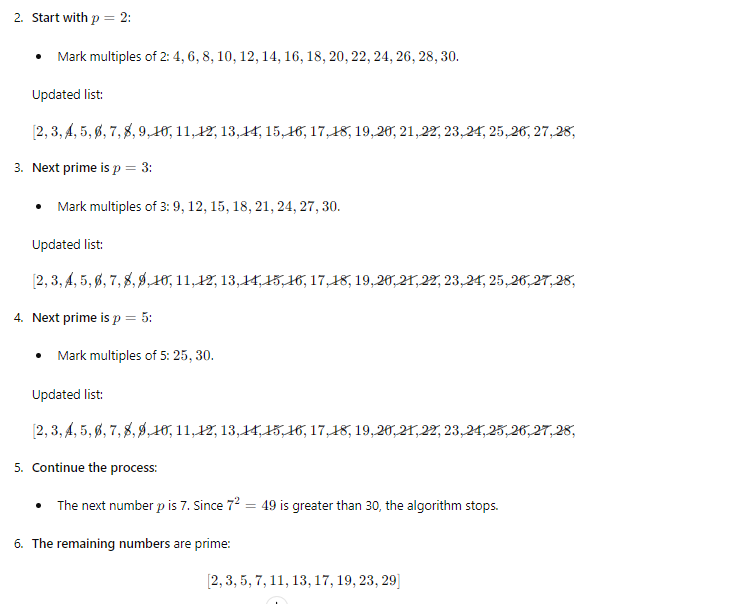
1. **Initialize** the first prime number, p=2.
2. **Mark the multiples of** p as non-prime (starting from p^2), since they are not prime.
3. **Find the next number** in the list that is still prime, set p to this new number, and repeat step 3.
4. **Continue the process** until p^2 is greater than n.
5. **All remaining numbers** in the list that are not marked are prime.

### Example: Finding Primes Up to 30

Let's find all the prime numbers up to n=30

1. **Create the list**:

[2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26,27,28,29,30]



In English

<https://www.geeksforgeeks.org/sieve-of-eratosthenes/>

In Polish

<https://www.algorytm.edu.pl/algorytmy-maturalne/sito-eratostenesa.html>

# Fast Exponentiation (Exponentiation by Squaring)

In English

<https://discuss.codechef.com/t/a-tutorial-on-fast-modulo-multiplication-exponential-squaring/2899?page=2>

<https://www.geeksforgeeks.org/exponential-squaring-fast-modulo-multiplication/>

<https://www.youtube.com/watch?v=WAzGvZbaAOw&ab_channel=SoftwareEngenius>

in Polish

<https://www.algorytm.edu.pl/algorytmy-maturalne/potegowanie-szybkie.html>

# Finding Roots - Newton-Raphson Method

In Polish

<https://www.algorytm.edu.pl/algorytmy-maturalne/newton-raphson.html>



# Finding the Minimum and Maximum Elements in a Set



# Additional content

<http://www.algorytm.org/>